Exercise 30

Find $f'(x)$ and $f''(x)$.

$$f(x) = \frac{x}{x^2 - 1}$$

Solution

Use the quotient rule to differentiate $f(x)$.

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(\frac{x}{x^2-1}\right) \\ &= \frac{\left[\frac{d}{dx}(x)\right](x^2-1) - \left[\frac{d}{dx}(x^2-1)\right](x)}{(x^2-1)^2} \\ &= \frac{(1)(x^2-1) - (2x)(x)}{(x^2-1)^2} \\ &= \frac{-x^2-1}{(x^2-1)^2} \end{aligned}$$

Use the quotient rule again to differentiate $f'(x)$.

$$\begin{aligned} f''(x) &= \frac{d}{dx}\left(\frac{-x^2-1}{x^4-2x^2+1}\right) \\ &= \frac{\left[\frac{d}{dx}(-x^2-1)\right](x^4-2x^2+1) - \left[\frac{d}{dx}(x^4-2x^2+1)\right](-x^2-1)}{(x^2-1)^4} \\ &= \frac{(-2x)(x^4-2x^2+1) - (4x^3-4x)(-x^2-1)}{(x^2-1)^4} \\ &= \frac{2x^5+4x^3-6x}{(x^2-1)^4} \\ &= \frac{2x(x^4+2x^2-3)}{(x^2-1)^4} \end{aligned}$$